

- **Iterated integrals:** evaluate an integral multiple times, treating other variables as constant in each step.

- **Double integrals:**

$$\iint_D f(x, y) dA = \lim_{(m,n) \rightarrow (\infty, \infty)} \sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*) \Delta A_{ij}$$

- Geometric meaning - volume under the surface  $f(x, y)$ .

- Area of a region in a plane:  $\iint_D dA$

- May be approximated using a double Riemann sum:

$$\iint_D f(x, y) dA \approx \sum_{i=1}^m \sum_{j=1}^n f(x_{ij}^*, y_{ij}^*) \Delta A_{ij}$$

- Average value of a function of two variables:  $= \frac{\iint_D f(x, y) dA}{\iint_D dA}$

- Flip order of integration or use change of variables if necessary.

- **Triple integrals:**

$$\iiint_E f(x, y, z) dV = \lim_{(l,m,n) \rightarrow (\infty, \infty, \infty)} \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n f(x_{ij}^*, y_{ij}^*, z_{ij}^*) \Delta V_{ij}$$

- Volume of a solid:  $\iiint_E dV$

- May be approximated using a triple Riemann sum:

$$\iiint_E f(x, y, z) dV \approx \sum_{i=1}^l \sum_{j=1}^m \sum_{k=1}^n f(x_{ij}^*, y_{ij}^*, z_{ij}^*) \Delta V_{ij}$$

- Average value of a function of three variables:  $= \frac{\iiint_E f(x, y, z) dV}{\iiint_E dV}$

- Change order of integration or use change of variables if necessary.

- **Fubini's Theorem:**

- For a double integral: If  $f(x, y)$  is continuous on the rectangle  $R = [a, b] \times [c, d]$ ,

$$\text{then } \iint_D f(x, y) dA = \int_a^b \int_c^d f(x, y) dy dx = \int_c^d \int_a^b f(x, y) dx dy .$$

- Can also be done with other iterated integrals.
- You can change the order of integration.

- Applications:

- Mass and total charge
- Centers of mass or centroids of laminas and solids